Homework 1

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1 Search vs decision

The P vs NP question is about decision problems. In this exercise, you will show that P=NP implies that certain search problems have efficient algorithms.

Show that if P=NP then there exists a polynomial-time algorithm that given a 3CNF formula φ outputs a satisfying assignment for φ if such an assignment exists.

2 Random subset sum

The set $\{0,1\}$ with addition and multiplication modulo 2 forms a finite field, denoted $\mathbb{F}_2$. The set $\{0,1\}^n$ with component-wise addition and scalar multiplication forms a vector space over this field, denoted $\mathbb{F}^2$. Just like for vector spaces over the reals, we can use matrices with their algebraic operations to describe linear maps on $\mathbb{F}_2^n$.

Show the following facts:

1. Let $x, y \in \mathbb{F}_2^n$ with $x \neq y$. Then,

$$\mathop{\mathbb{P}}_{r\in\mathbb{F}_2^n}\left\{r^T x \neq r^T y\right\} = 1/2\,. \tag{1}$$

2. Let $A \in \mathbb{F}_2^{n\times n}$ be an invertible matrix and let $y \in \mathbb{F}_2^n$. Then,

$$\mathop{\mathbb{P}}_{x\in\mathbb{F}_2^n}\left\{Ax = y\right\} = 2^{-n}\,. \tag{2}$$

3. Let $x, y \in \mathbb{F}_2^n$ with $x \neq 0$. Then,

$$\mathop{\mathbb{P}}_{A\in\mathbb{F}_2^{n\times n}}\left\{Ax = y\right\} = 2^{-n}\,. \tag{3}$$

3 Gap preserving reduction for independent set

In the lecture on 2/9, we sketched a gap preserving reduction from 3Sat to independent set. This exercise asks you to fill in the details for the proof sketch of the lemma.

1. Describe the polynomial-time function f that maps every max3sat instance φ to independent set instance G.
2. Describe how every assignment x to the max3sat instance φ corresponds to an independent set S in G.
3. Describe how every independent set S in G corresponds to an assignment x for φ.

4 Approximation algorithm for MaxQuadEq

In this exercise, you are to fill in details of the approximation algorithm for MaxQuadEq mentioned in the lecture on 2/11.

1. Show that a random assignment satisfies in expectation at least 1/4 of the equations of a satisfiable system of quadratic equations.
2. Develop a randomized polynomial-time algorithm that outputs an assignment which satisfies in expectation at least 1/4 of the equations satisfied by an optimum assignment.

Footnotes