

# Homework 4

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## 1 Spectral gap under addition

In the exercise, you are to show that adding an expander to an arbitrary graph yields an expander.

Let  $G_1$  and  $G_2$  be  $d$ -regular graphs with the same set of vertices  $V$ . Construct a  $2d$ -regular graph  $H$  by adding the edges of  $G_2$  to  $G_1$ . Note that the normalized adjacency matrix of  $H$  satisfies  $A_H = (A_{G_1} + A_{G_2})/2$ .

Suppose  $G_2$  satisfies  $\lambda(G_2) \leq 1 - \varepsilon$  for some  $\varepsilon > 0$ . Show that then  $H$  satisfies  $\lambda(H) \leq 1 - \varepsilon/2$ .

## 2 Combinatorial characterization of positive spectral gap

Show that a regular graph  $G$  is connected and non-bipartite if and only if  $\lambda(G) < 1$ .

## 3 Combinatorial expansion of random regular graphs

Do Exercise 21.11 in the textbook.

## Footnotes